Channel Geometry

A channelized stream in poor condition

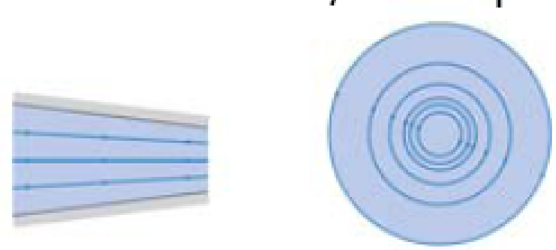
Types of flow:

- **Uniform**:
  - velocity is constant along the streamline
  - Streamlines are straight and parallel

(a) (b)

$$\frac{\partial V}{\partial s} = 0$$

- **Non-uniform**:
  - velocity changes along streamline
  - Streamlines are curved and/or not parallel



$$\frac{\partial V}{\partial s} \neq 0$$

Slide from Dr. Isaac

Types of flow:

- **Steady**:
  - streamline patterns are not changing over time

$$\frac{\partial V}{\partial t} = 0$$

- **Unsteady**:
  - velocity at a point on a streamline changes over time

$$\frac{\partial V}{\partial t} \neq 0$$

Unsteady Flow

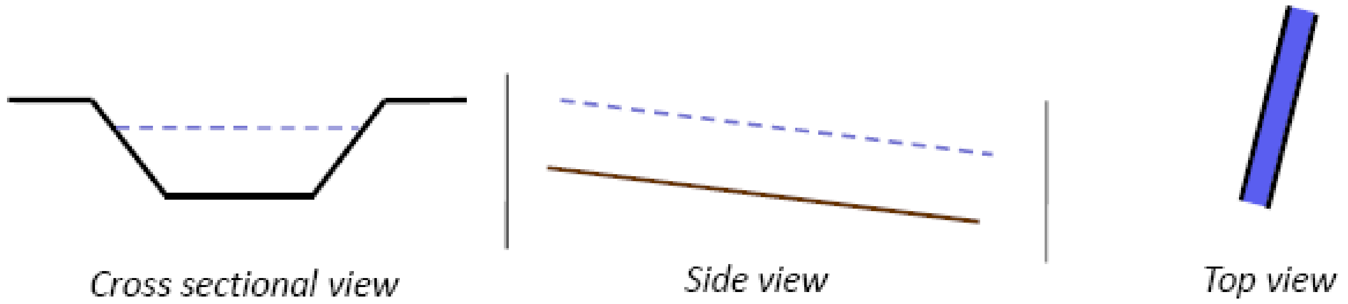
t = 0

t > 0

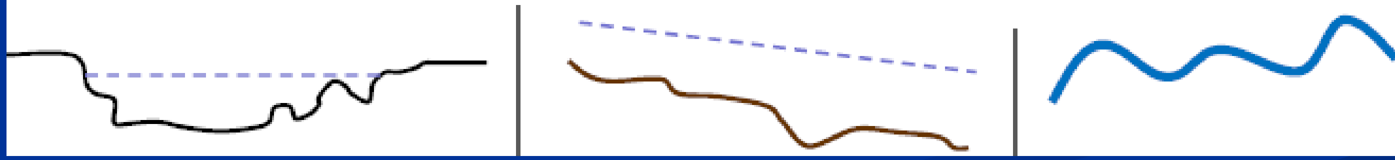
Slide from Dr. Isaac

Channel Types

- Prismatic:
  - Cross sectional shape is constant
  - Slope of channel is constant
  - Alignment of channel is straight (top view)



- Non-prismatic:
  - Variations in shape, slope, and alignment



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Channel Geometry

Channel geometry encompasses all characteristics that define the channel geometry

At a point, it includes

area, depth, friction, bottom width, top width, side slope, wetted perimeter.

For the entire channel reach, it includes

length, slope, average of each of the first 5 parameters

A concrete trapezoidal channel

Channel Geometry

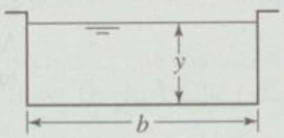
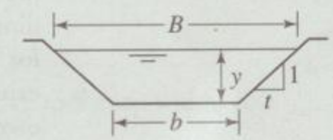
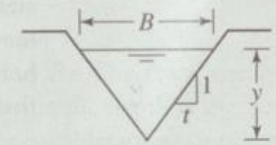
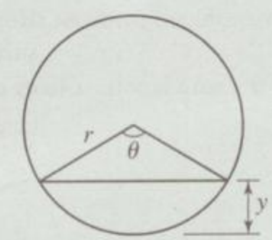
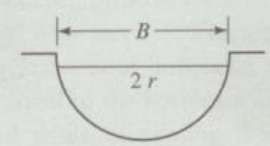
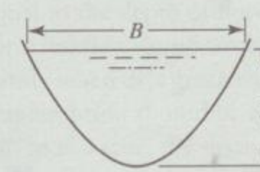
For artificial channels

D: hydraulic depth

D = A / B

A: area
B: top width

TABLE 10.1 Properties and Geometric Elements of Typical Channel Cross Sections

Section	Area, A	Wetted Perimeter, P	Hydraulic Radius, R	Top Width, B	Hydraulic Depth, D	Cross Section
Rectangular	by	$b + 2y$	$by/(b + 2y)$	b	y	
Trapezoidal	$(b + ty)y$	$b + 2yw$, $w = (1 + t^2)^{0.5}$	A/P	$b + 2ty$	A/B	
Triangular	ty^2	$2yw$	$ty/(2w)$	$2ty$	A/B	
Circular	$(\theta - \sin\theta)\frac{d^2}{8}$	$r\theta$	$\left(1 - \frac{\sin\theta}{\theta}\right)d/4$	$2r\sin(\theta/2)$	A/B	 $\theta = 2\cos^{-1}\left(1 - 2\frac{y}{d}\right)$
Semicircular	$\pi r^2/2$	πr	$r/2$	$2r$	$\pi r/4$	
Parabolic Section	$2/3By$	$B + (8/3)y^2/B^*$	$2B^2y/(3B^2 + 8y^2)$	$3A/(2y)$	$2/3y$	

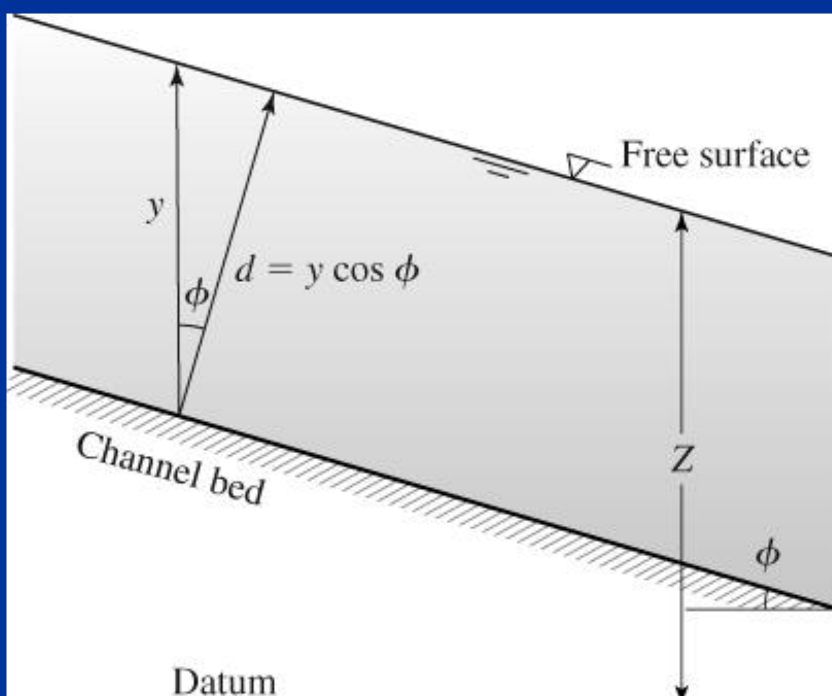
*Approximation for the interval $0 < \frac{4y}{B} < 1$.

Geometric Elements

Channel cross-sectional shape governs the pattern of the velocity distribution, boundary shear stress & secondary circulation => DISCHARGE CAPACITY

y (depth of flow): the vertical distance of the lowest pt of a channel section to free surface

See the text book for the definition of other elements.



Depth of flow (y) and depth of flow section (d)

(Chapter 11)
Resistance in Open Channels

Steady, uniform flow in a well-maintained channel

prepared by Ercan Kahya

Governing Equations in Open Channel Flow

- Continuity:

$$Q_1 = V_1 A_1 = V_2 A_2 = Q_2$$

- Momentum:

Darcy-Weisbach

$$V = \sqrt{\frac{8g}{f} R S_f}$$

Manning / Strickler

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

Uniform flow: $S_f = S_0 = S$

- Energy:

$$\left(y_1 + \alpha_1 \frac{V_1^2}{2g} + z_1 \right) = \left(y_2 + \alpha_2 \frac{V_2^2}{2g} + z_2 \right) + h_f$$

Slide from Dr. Isaac

Manning Equation

Empirical equation where friction factor "n" does not account for flow conditions

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

V = flow velocity [=] m/s

R = hydraulic radius = A/P_w [=] m

S = channel slope

n = Manning friction factor

Material	n
Lined Channels:	
Asphalt	0.013 – 0.017
Brick	0.012 – 0.018
Concrete	0.011 – 0.020
Rubble or riprap	0.020 – 0.035
Vegetal	0.030 – 0.40
Excavated or dredged channels:	
Earth, Straight and uniform	0.020 – 0.030
Earth, winding, fairly uniform	0.025 – 0.040
Rock	0.030 – 0.045
Unmaintained	0.050 – 0.14
Natural Channels: (width < 31 m)	
Fairly regular section	0.03 – 0.07
Irregular section with pools	0.04 – 0.10

Slide from Dr. Isaac

Channel Efficiency

- Capacity (efficiency) varies inversely with the wetted perimeter (P).

- Energy losses are less in channels with smaller P & vice versa.

- 4 options to excavate a rectangular section with an area of 20m2 :

- P: (a) 22; (b) 14; (c) 13; and (d) 14m → Choice is "c"

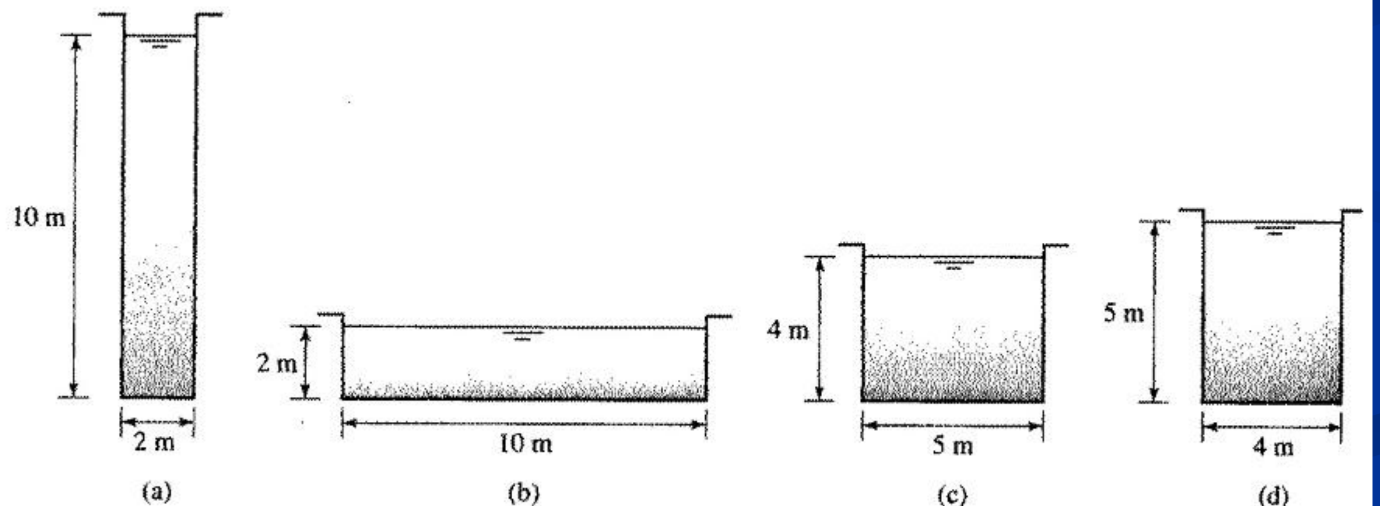


Figure 11.6 Some options for the dimensions of a 20 m² rectangular section.

Channel Efficiency

- For Max. Capacity Hydraulic Efficiency → Min. wetted perimeter (P).

- That section is called "*most efficient*" or "*best hydraulic*" section.

- It reduces the cost of lining.

- For a given cross-sectional area: the best hydraulic section has min P

- For a given perimeter: the best hydraulic section has max A (area).

For trapezoidal sections:

Area → $A = by + ty^2$

Wetted perimeter → $P = b + 2y(1 + t^2)^{1/2}$

Channel Efficiency

Determine the ***relation btw b & y*** to minimize P for a fixed cross-sectional area and side slope

$$b = \frac{A - ty^2}{y}$$

Substitute into wetted perimeter eq.:

$$P = \frac{A - ty^2}{y} + 2y(1 + t^2)^{1/2}$$

To minimize P w.r.t. *y:*

$$\frac{dP}{dy} = 0$$

yields

$$b = 2y[(1 + t^2)^{1/2} - t]$$

Note that this eq. leads to impractical design, such as very deep & narrow channel.

Channel Efficiency

- Alternatively "b" equation can be written as:

$$b + 2ty = 2y\sqrt{1 + t^2}$$

This implies that B (Top width) = 2L (side length)

Referring this figure

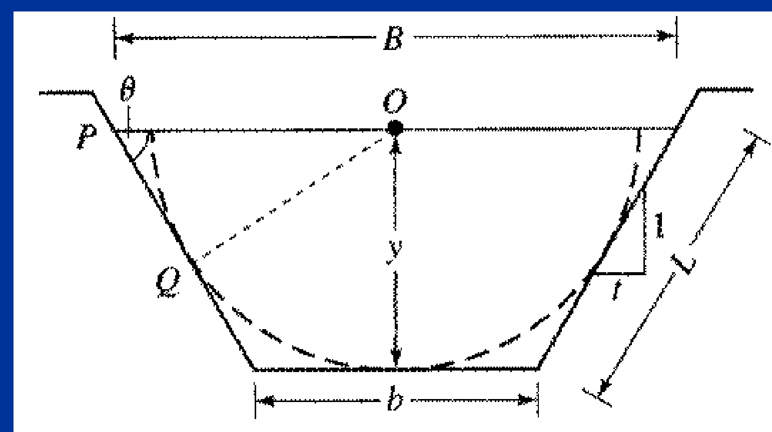


Figure 11.7 A best hydraulic trapezoidal section.

$$OQ = OP\sin\theta$$

$$= \frac{B}{2}\frac{1}{\sqrt{1 + t^2}} = \frac{b + 2ty}{2}\frac{1}{\sqrt{1 + t^2}}$$

Channel Efficiency

- Substitute the final form of '*b*' equation into *OQ* relation:

$$OQ = \frac{2y[\sqrt{1+t^2} - t] + 2ty}{2} \frac{1}{\sqrt{1+t^2}} = y$$

Thus, a semicircle with its center at *O* coinciding with the channel axis and of radius *y* can be drawn tangential to the bed and sides as shown in Figure 11.7.

Moreover minimization w.r.t. 't" of side slope yields

$$t = \frac{1}{\sqrt{3}}$$

This corresponds to a *slope angle of 60* degree

But this slope is often too steep for natural channels

- Substitute this again into the final form of '*b*' equation :

$$b = 1.155y$$

⟺ This defines the trapezoidal section of greatest possible efficiency

Channel Efficiency

- Even if we further substitute $b = 1.155y$ into R equation of trapezoidal section:

$$R = \frac{y}{2}$$

$$R = \frac{A}{P} = \frac{by + zy^2}{b + 2y(1 + z^2)^{1/2}}$$

hydraulic radius equal to one-half the normal depth,

In the case of rectangular channels ($z = 0$)

$$b + 2zy = 2y\sqrt{1 + z^2}$$

reduces to

$$b = 2y$$

The most efficient rectangular channel is one in which *the depth is one-half of the width.*

Channel Efficiency

Example 11.5

A trapezoidal channel is to be designed to carry a discharge of 150 m^3/s and run on a slope 0.0025 m/m with side slopes of $2H/1V$. If the channel is to be designed for maximum hydrau efficiency (subject to the side slope restriction), what would be the depth and width? Let Manning n value be 0.035.

Solution:

$$b = 2y[(1 + t^2)^{1/2} - t]$$

For $t = 2$, $b = 0.472y$. Then, from Manning's equation,

$$AR^{2/3} = \frac{nQ}{S_o^{1/2}} = \frac{0.035 \times 150}{(0.0025)^{1/2}} = 105$$

or

$$(by + ty^2)\left[\frac{by + ty^2}{b + 2y(1 + t^2)^{1/2}}\right]^{2/3} = 105$$

but since $b = 0.472y$,

$$2.472y^2(0.5y)^{2/3} = 105$$

$$y = 4.85 \text{ m}$$

and

$$b = 0.472y = 2.29 \text{ m}$$

Resistance in Steady Nonuniform Flow

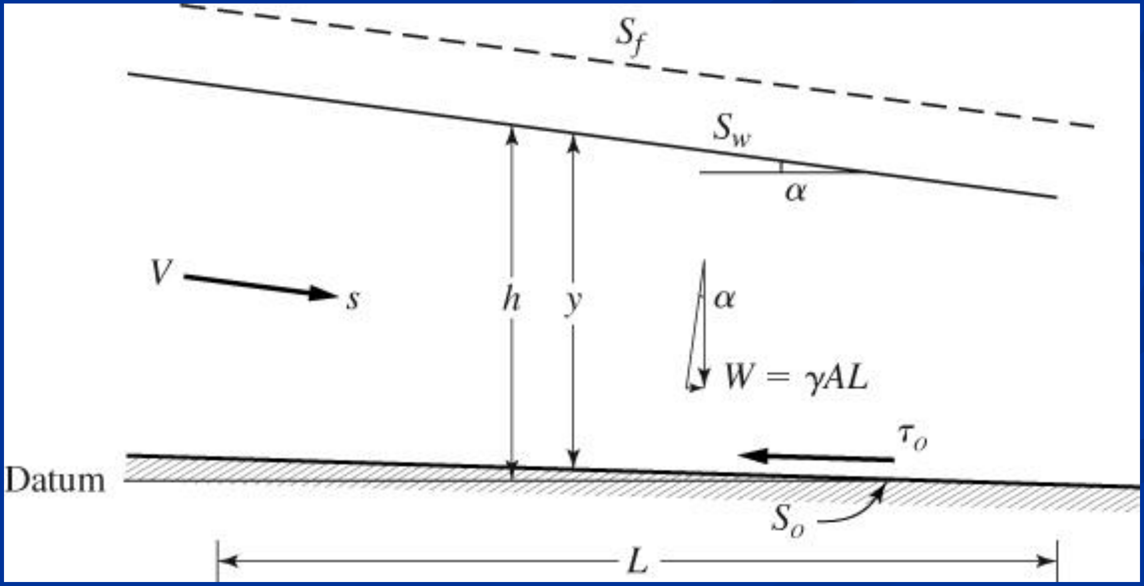
- Manning and Chezy equations can be generalized to "**nonuniform flow**"

- **Assumption**: the stage changes only gradually w.r.t. longitudinal distance

- Then this flow is called ***GRADUALLY VARIED FLOW***

- But acceleration is not negligible! Makes it different than uniform flow

- *Channel slope, water surface,* and *energy gradient* are not equal!



Resistance in Steady Nonuniform Flow

Applying Newton`s second law:

$$\gamma AL \sin\alpha - \tau_o LP = \rho ALa_s$$

the subscript s denotes the direction of flow.

In the case of steady flow, only convective term $v\frac{\partial v}{\partial s}$ needs to be considered

After math manipulations;

$$\tau_o = \gamma R \sin\alpha - \gamma R\left(\frac{v}{g}\right)\frac{\partial v}{\partial s}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = S_w$$

S_w is the slope of the water surface.

$$\sin\alpha = S_w \cos\alpha$$

Shear stress at the bed;

$$\tau_o = \gamma RS_w \cos\alpha - \gamma R\left(\frac{v}{g}\right)\frac{\partial v}{\partial s}$$

Resistance in Steady Nonuniform Flow

Now, noting that $S_w = -dh/ds$, and the cos of small angles approaches 1

$$\tau_o = -\gamma R\left(\frac{dh}{ds} + \frac{v}{g}\frac{\partial v}{\partial s}\right)$$

or

$$\tau_o = -\gamma R\frac{d}{ds}\left(h + \frac{V^2}{2g}\right)$$

Note that

$$\frac{d}{ds}\left(h + \frac{V^2}{2g}\right) = S_f$$

FRICTION SLOPE

$$\tau_o = \gamma R S_f$$

FLOW IS BEING DRIVEN BY THE HYDRAULIC GRADIENT

That is to say; the component of gravity force in the flow direction

Class Exercises:

11.1. Water is flowing in a trapezoidal earthen channel width 2 m and side slopes 1.5 *H*/1 *V*. The channel is carrying a discharge of 50 m^3/s and is running on a slope of 0.0025 m/m. If the roughness coefficient is 0.030, what is the normal depth in the channel?

11.15. It is desired to design a trapezoidal channel with a bottom width of 10 ft and 2 *H* on 1 *V* side slope. Sieve analysis revealed a grain size distribution, which result in an allowable bed shear stress of 0.5 lb/ft^2 and a Manning n value of 0.03. If the channel is to be designed to run at normal depth of 5 ft, what will be the resulting discharge?

11.17. A 20 ft wide rectangular channel carries a discharge of 400 ft^3/s at a normal depth of 10 ft. If the roughness coefficient is 0.03, what shear stress in lb/ft^2 is imparted to the channel boundary by this flow?